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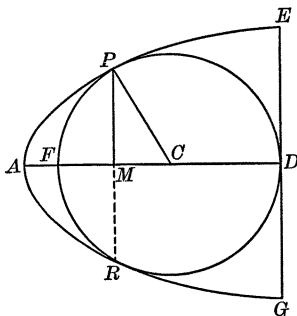
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Let $PFRD$ be the given circle whose center C is the origin of rectangular coördinates. Then, without loss of generality, the parabola may be assumed to have its axis on the axis of abscissas AD .

Let EAG be the required parabola. Then, if r is the radius of the circle, we have $x^2 + y^2 = r^2$ for the circle, and $y^2 = 2p(x + k)$ for the parabola, where p is the distance from the focus to the directrix and k is the distance AC .

Since P is the point of tangency, solving these two simultaneous equations subject to the condition of tangency, we have $r^2 - 2kp + p^2 = 0$.



Hence, $k = p/2 + r^2/2p$.

Then

$$AD = AC + CD = \frac{p}{2} + \frac{r^2}{2p} + r = \frac{(r + p)^2}{2p},$$

and

$$ED = \sqrt{2p(AD)} = (r + p).$$

Area of $EAG = a = \frac{2}{3}AD \cdot EG = \frac{4}{3}(r + p)^2/2p \cdot (r + p) = \frac{2}{3}(r + p)^3/p$.

Equating to zero the derivative of a with respect p , we have

$$\frac{da}{dp} = \frac{2}{3} \cdot \frac{3p(r + p)^2 - (r + p)^3}{p^2} = 0;$$

whence

$$(p + r)^2 = 0 \quad \text{and} \quad 2p - r = 0.$$

Hence, $p = -r$ or $p = \frac{1}{2}r$. The value $p = -r$ gives neither a maximum nor a minimum. The value $p = \frac{1}{2}r$ gives a minimum, and the equation of the corresponding parabola is $y^2 = r(x + 5r)/4$.

A similar solution was received from the PROPOSER.

359. Proposed by W. D. CAIRNS, Oberlin College.

Examine for maxima and minima

$$f(x) = e^{-cx}(1 + \cos x) \quad (c > 0)$$

SOLUTION BY A. M. HARDING, University of Arkansas.

$$f'(x) = -e^{-cx}(c + c \cos x + \sin x).$$

$$f''(x) = e^{-cx}(c^2 + c^2 \cos x + 2c \sin x - \cos x).$$

Now $f'(x) = 0$ only when $c + c \cos x + \sin x = 0$,
that is, when

$$c(1 + \cos x) + \sin x = 0,$$

or

$$2c \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2} = 0.$$

Hence,

$$\cos \frac{x}{2} = 0, \quad \text{and} \quad x = \pi, 3\pi, 5\pi, \dots (2n-1)\pi, \dots$$

or

$$c \cos \frac{x}{2} + \sin \frac{x}{2} = 0, \quad \text{and} \quad x = 2 \arctan(-c).$$

When $x = \pi, 3\pi, 5\pi, \dots, (2n-1)\pi, \dots$, $f''(x) = -c(2n-1)\pi$, which is always positive.

Hence, the minimum value of $f(x)$ is obtained by giving x any of these values. When $x = 2 \arctan(-c)$, that is, $c \cos x/2 + \sin x/2 = 0$,

$$\begin{aligned} f''(x) &= e^{-cx}[c^2(1 + \cos x) + c \sin x - \cos x] \\ &= e^{-cx} \left[2c \cos \frac{x}{2} \left(c \cos \frac{x}{2} + \sin \frac{x}{2} \right) + 2c \sin \frac{x}{2} \cos \frac{x}{2} - \cos x \right] \\ &= e^{-cx} \left[2c \sin \frac{x}{2} \cos \frac{x}{2} - \cos x \right] = e^{-cx} \left[2 \sin \frac{x}{2} \left(-\sin \frac{x}{2} \right) - \cos x \right] \\ &= -e^{-cx} \left[2 \sin^2 \frac{x}{2} + \cos x \right] = -e^{-cx}[1 - \cos x + \cos x] = -e^{-cx}. \end{aligned}$$

Now $-e^{-cx}$ is negative for the above value of x . Hence the maximum value of $f(x)$ is obtained by giving x the value $2 \arctan(-c)$.

Also solved by W. C. EELLS, PAUL CAPRON, H. C. FEEMSTER, G. W. HARTWELL and the PROPOSER.

MECHANICS.

286. Proposed by C. N. SCHMALL, New York City.

A slightly elastic string is just long enough to reach between two hooks on the same horizontal line. A ring of weight w is placed at its middle point. Show that the ring will sink through a distance $h = a\sqrt{3e\omega/2}$, where e is the elasticity of the string and $2a$ the distance between the two hooks.

SOLUTION BY B. F. FINKEL, Drury College.

Since w is at the middle point of the string, the tension T in the two halves of the string is the same when the string is in equilibrium. Let θ be the angle which the string makes with the horizontal line.